River discharge and bathymetry estimations from SWOT measurements

K. Larnier, J. Monnier, P.-A. Garambois, J. Verley

Objectives & Results

- **Goal** An algorithm to estimate the discharge of rivers observed by the forthcoming SWOT mission (NASA-CNES et al. 2021)
- **Elaborated Algorithm** The **Hierarchical Variational Discharge Identification (HiVDI)** algorithm. (Open-source software DassFlow).
- **Capabilities** Estimations of the three key flow features : an effective bathymetry $b(x)$, a roughness coefficient K law, the discharge $Q(t)$ (at the observation period).
- **Ingredients** Advanced Variational Data Assimilation (VDA) formulation applied to the Saint-Venant equations combined with low complexity algebraic systems.

Flow models

- Saint-Venant's equations (1D shallow-water).

$$\begin{cases} \partial_t A + \partial_x Q & = 0 \\ \partial_t Q + \partial_x \left(\frac{Q^2}{A}\right) + gA\partial_x Z & = -gA\, S_f \end{cases}$$

with $S_f \equiv S_f(A, Q; K) = \frac{|Q|Q}{K^2 A^2 R_h^{4/3}}$. Imposed B.C. : $Q_{in}(t)$ at inflow and normal depth at outflow. Strickler K is reach (r) dependent : $K_r(h) = \alpha_r\, h^{\beta_r}$.

- Algebraic systems : low-complexity model

Steady-state, low Froude assumptions : "0.5D"

$$D_c \cdot (\tilde{K}_{r,p} A_{r,0})_{R\times P} \; + \; D_d \cdot \tilde{K}_{RP} \; = \; \tilde{Q}_{RP}$$

with : $\tilde{K}_{RP} = (K_{r,p}^{3/5})_{r,p} \in R^{RP}$, $A = (A_{r,0})_{r,} \in R^{R}$
$\tilde{Q}_{RP} = (Q_{r,p}^{3/5})_{r,p} \in R^{RP}$.

- River description from SWOT measurements.

R reaches ($\approx$ 200 m long, RiverObs), P overpasses.

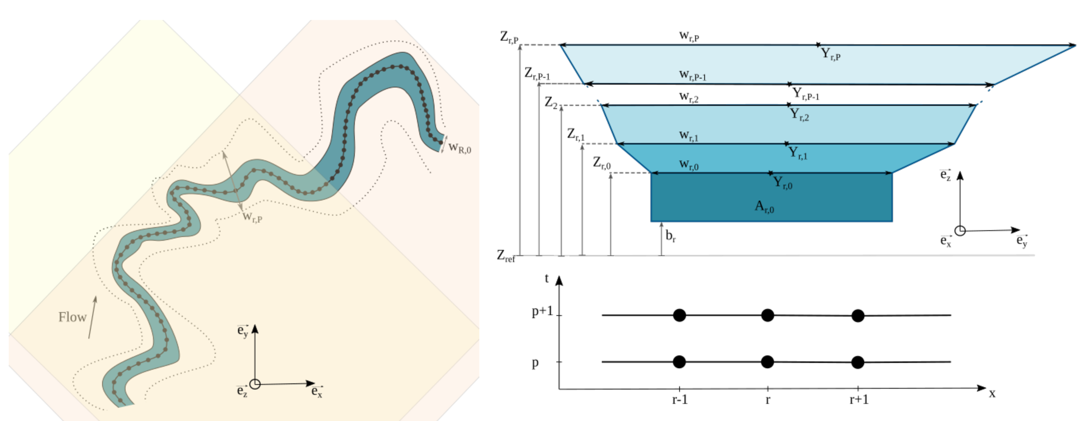


FIGURE – (Right Top) Effective river cross section at reach r defined from SWOT data set $\{Z_{r,p}, W_{r,p}\}_{R,P+1}$. (Right Bottom) Space - time stencil (r,p). x denotes the curvilinear abscissa along the river center line defined at low flow by $Y_{r,0}$ with $Y_{r,p}$ the middle of the cross sectional width.

VDA formulation

- Minimisation formulation

$$\min_k J(k) \text{ with } k = B^{-1/2}(c - c_{prior})$$

c vector of the unknown "parameters" (control var.) :

$$c = (\{Q_{in}\}_{1..P};\ \{b\}_{1..R};\ \{\alpha, \beta\}_{1..R}) \in \mathbb{R}^{P+2R}$$

$B = diag(B_Q, B_b, B_K)$ covariance matrices (2nd order auto-regressive operators) with (prior) length scales of correlation ($\sim$ prior probabilistic model).

- Cost function

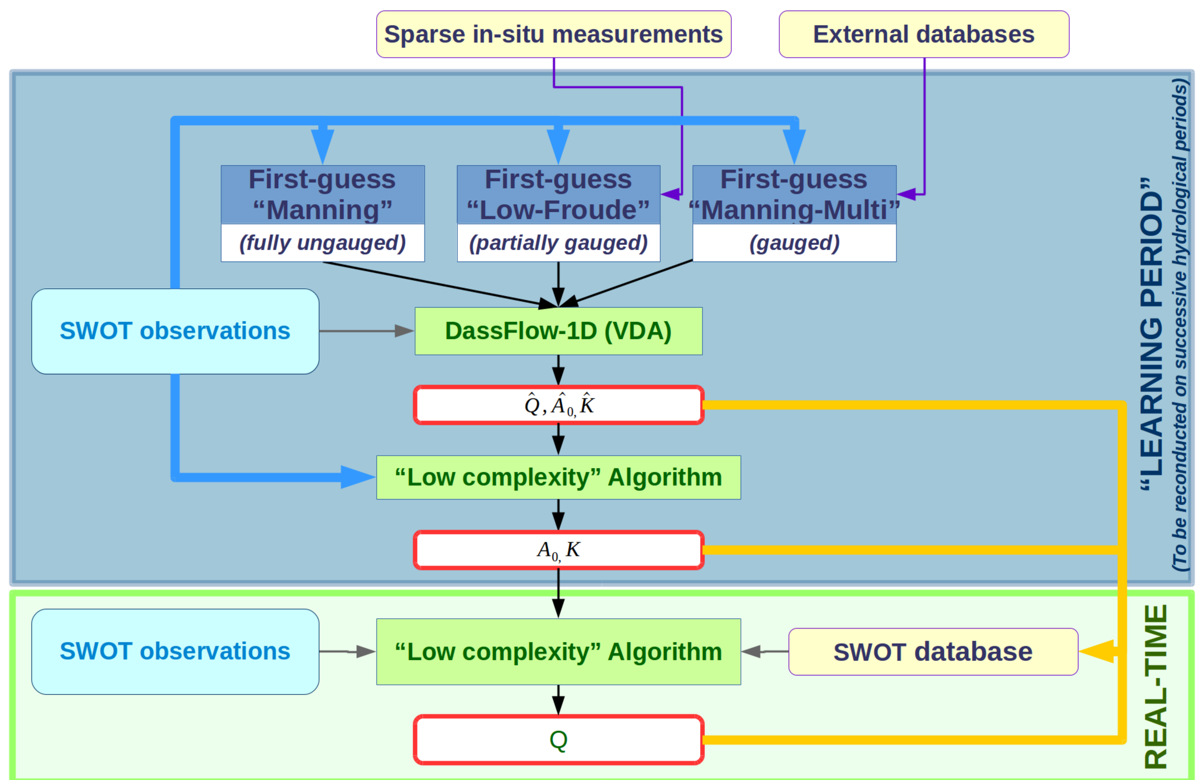
$$J(k) = j(c) \; = \; \|Z(c) - Z^{obs}\|_N^2 \; + \; \gamma_{reg}\, j_{reg}(c)$$

$\Rightarrow$ misfit between the model output and the altimetry measurements in adequate metrics (prior proba models).

- Gradient-based optimisation with the gradient computed from the *adjoint model* obtained by Algorithmic Differentiation of the direct code.

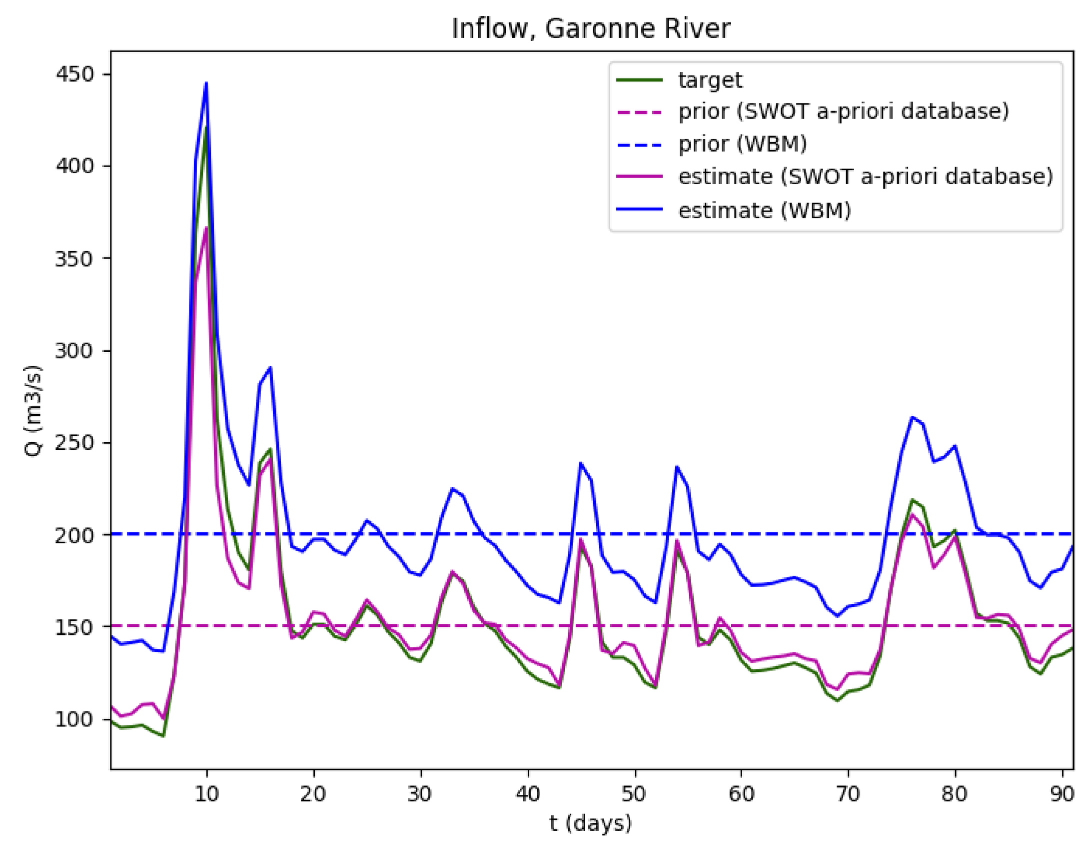
The HiVDI algorithm

- Estimation of rivers discharge in two stages :
  - Calibration (1 year, VDA processes)
  - Real-time estimations (low-complexity/0.5D model))

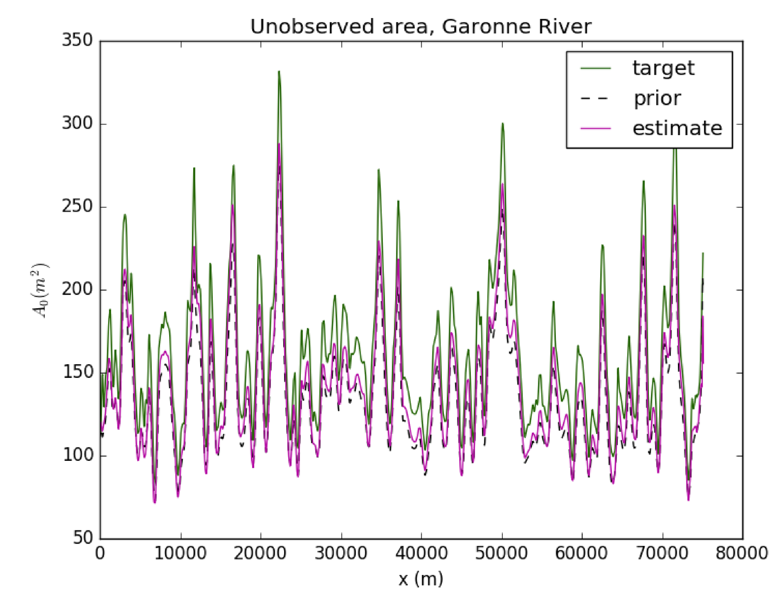


Numerical tests

- **Garonne River (synthetic data with noise)**

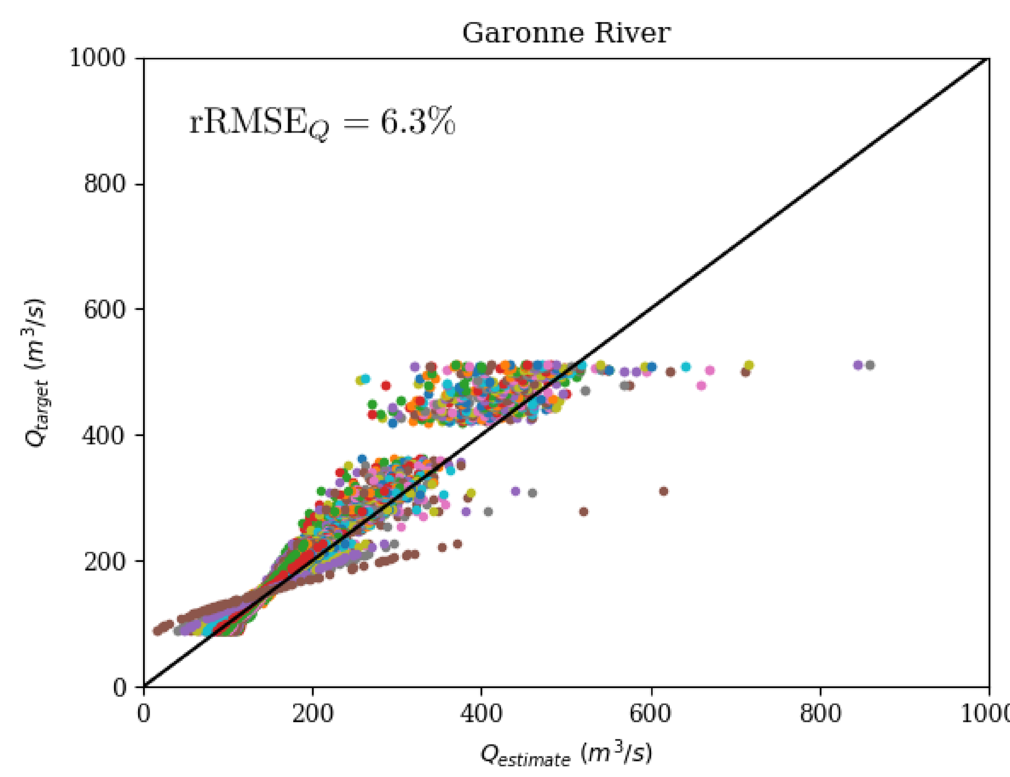


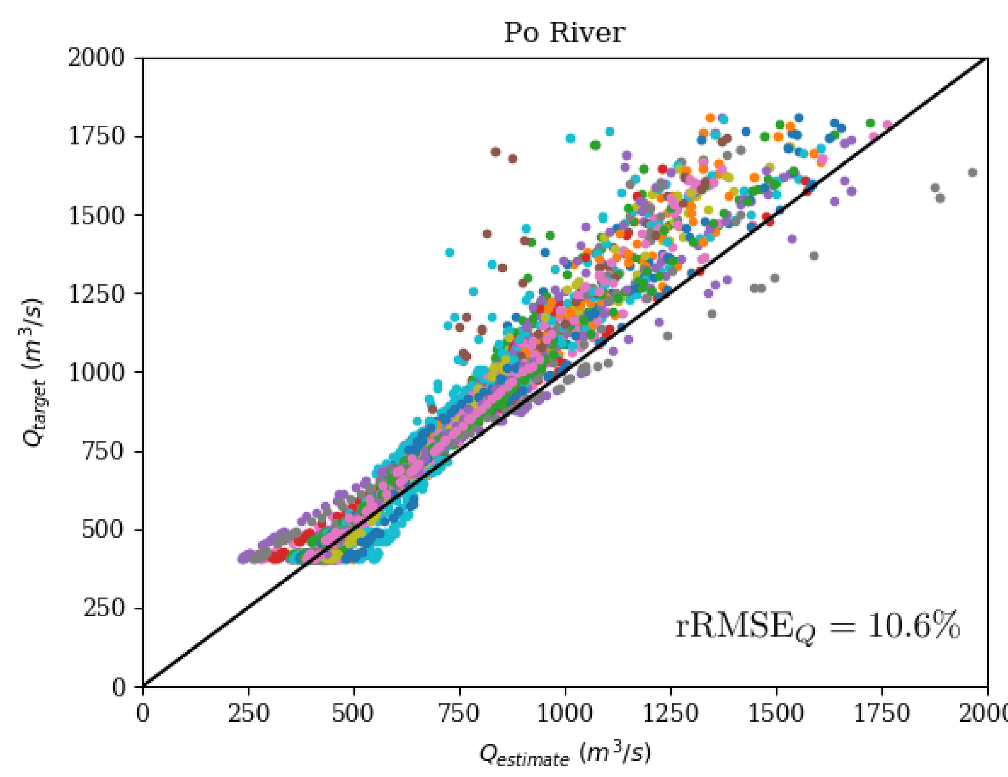
- Time window : 90 days
- Length : 76 km
- RiverObs nodes ($\approx 200m, 1day$)
- Twin-experiment, $\epsilon_Z \sim \mathcal{N}(0, 25cm)$



- Space-time variations of the flow hydrograph accurately retrieved
- However potential shift depending to the first-guess $Q^{(0)}$

- **Assessment of the low-complexity model (0.5D)**
  1. Calibration of the algebraic system using results from the VDA
  2. Validation on the remaining observations (9 months) :





- Good accuracy of the infered discharge values (after using the complete toolchain VDA + 0.5D model)
- This low complexity model enables real-time estimations (0.5 μs / reach / pass)

References

K. Larnier, J. Monnier, P.-A. Garambois, J. Verley. "Estimation of rivers discharge from altimetry". Revised version submitted.
P. Brisset, J. Monnier, P.-A. Garambois, H. Roux. "On the assimilation of altimetry data in 1D Saint-Venant river models". Adv. Water Ress., vol. 119, pp 41-59, 2018

Conclusions

- Robust hierarchical 0.5D-1D modeling.
- Priors may be obtained from ancillary databases & the low-complexity model.
- Real-time estimations possible passed a 1 year calibration period (low-complexity model).
- Use of AirSWOT and in-situ data (not shown).

Perspectives

- Investigate thoroughly how to define better priors.
- Coupling hydrology model with 1D hydraulics and 2D local finer models.

Acknowledgements: The authors K. Larnier (software engineer at CS corp.) and J. Verley (software engineer during 10 months next PhD student IMT-INSA-CLS 17-18) have been funded by CNES. The two other authors have been partly supported by CNES TOSCA research project 14-18.
DassFlow computational software, open source upon request: http://www/math.univ-toulouse.fr/DassFlow.